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Summing Planar Bosonic Open Strings [1]

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Abstract

In earlier work, planar graphs of massless ϕ^3 theory were summed with the help of the light cone world sheet picture and the mean field approximation. In the present article, the same methods are applied to the problem of summing planar bosonic open strings. We find that in the ground state of the system, string boundaries form a condensate on the world sheet, and a new string emerges from this summation. Its slope is always greater than the initial slope, and it remains non-zero even when the initial slope is set equal to zero. If we assume the initial string tends to a field theory in the zero slope limit, this result provides evidence for string formation in field theory.

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1. Introduction

A couple of years ago, the present author and Charles Thorn initiated a program of studying field theory in the planar limit by reformulating it as a local theory on the world sheet [1]. This new formulation provided a fresh approach for tackling some of the old standing problems. The field theory most intensively studied so far is massless ϕ^3 theory [2-6], although Thorn and collaborators later extended the world sheet approach to more realistic models [7-9]. Apart from providing a new insight into field theory, the world sheet formulation enables one to do dynamical calculations, using the mean field approximation. The most interesting result so far to come out of the mean field method was the emergence of a string picture from the sum of the planar graphs in the ϕ^3 theory. Of course, there are various caveats: The model is unphysical and in fact unstable, and the reliability of the mean field approximation is open to question. There are also various technical problems which were only partially overcome in [6]. In spite of all the drawbacks, we feel that an important step forward has been taken.

In this article, instead of summing field theory diagrams, we consider the sum of planar bosonic open string diagrams, and we show that the mean field method is also applicable to this case. We list below the main motivations for this generalization:

1) Some of the technical problems encountered in summing field theory diagrams are absent in the case of the string diagrams. In fact, summing strings turns out to be simpler than summing ϕ^3 graphs.

2) Assuming that the zero slope limit of the string theory is some field theory, one can indirectly recover the field theory sum from the string sum by taking the zero slope limit.

3) The string sum is of interest in itself; it may enable one to investigate problems such as tachyon condensation [10].

The main results to emerge from this investigation are the following: After the summation, a new string emerges, whose slope is greater than the original slope. The dynamical mechanism for this change is what we call the condensation of the string boundaries. What happens is that the boundaries become dense on the world sheet, changing its texture. This phenomenon was already observed in the context of the ϕ^3 theory [2-6]; in fact, in this respect, the string and field theory calculations are remarkably similar. The crucial point is that even after the initial slope is set equal to zero, the final slope after the summation remains finite. Since the zero slope limit of string

theory is generally believed to be a field theory, this result supports the idea of string formation in field theory.

All of this, of course, depends crucially on the validity of the mean field approximation. In addition, there are some questions on the meaning of the zero slope limit. Usually, in taking the zero slope limit, the vector meson is kept at zero mass, and the heavy particles decouple. The resulting field theory is therefore a vector (gauge) theory. The existence of the tachyon, however, throws some doubt on this picture; in this limit, the tachyon becomes infinitely heavy and therefore infinitely destabilizing. Clearly, it is desirable to generalize the approach developed in this paper to the tachyon free superstring theory, where this problem is absent. The present article can be thought of as a warmup exercise for this future project.

In section 2, we briefly review the Feynman graphs in the mixed lightcone variables [11] and the and the local field theory on the world sheet which generates these graphs [1-3]. We also discuss the transformation properties of various fields under a special Lorentz boost, which manifests itself as a scale transformation on the world sheet.

The technology introduced in section 2 for summing over field theory graphs will turn out to be exactly what is needed later on for summing over planar strings in section 3. As it turns out, the action on the world sheet that reproduces the string sum can be cast in a form very similar to the corresponding action for field theory by means of a duality transformation. This action is then the starting point of the mean field method developed in section 4 from the point of view of the large D limit, where D is the dimension of the transverse space. Part of this section is in the nature of a review, since there is a lot in common here with references [2-6], where the mean field method was applied to the ϕ^3 field theory. The section ends with a discussion of how to define cutoff independent parameters from the cutoff dependent ones.

In section 5, the mean field method is applied to the calculation of the ground state of the model. The equations determining the ground state have two possible solutions, which we call the $(+)$ and $(-)$ phases. The $(+)$ phase describes the original perturbative sum of the strings. The $(-)$ phase, which has lower energy and therefore is the true ground state, is the phase where the string boundaries have formed a condensate on the world sheet. We show that, as a result of this condensation, a new string is formed, with a slope greater than the slope of the original string. This new slope remains non zero even when the initial slope is set equal to zero. Identifying the zero

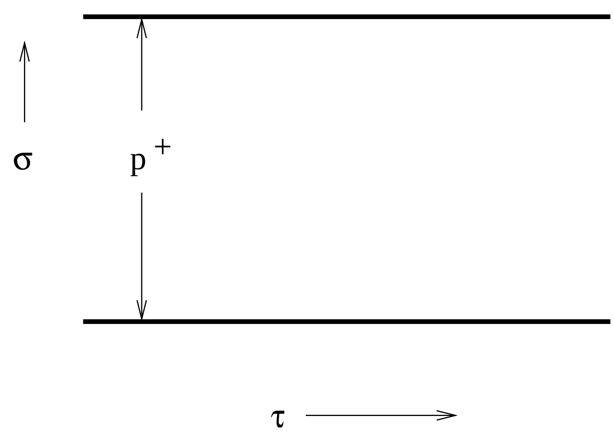

Figure 1: The propagator

slope limit of the string with field theory, we consider this result as a strong indication of string formation in field theory.

Finally, we discuss our results in section 6, and summarize our conclusions and point out some future directions of research in section 7.

2. A Brief Review

In this section, we present a brief review of the results obtained in references [1-6]. In this work, starting with the world sheet representation of the sum of the planar graphs of the massless ϕ^3 field theory [1-3], the ground state energy of the system was calculated in the mean field approximation. It was found that, subject to this approximation, the dynamics favors string formation.

The starting point of the mean field calculation is the light cone representation of the scalar propagator [11]

$$\Delta(p) = \frac{\theta(\tau)}{2p^+} \exp\left(-i\tau\, \frac{\mathbf{p}^2 + m^2}{2p^+}\right), \tag{1}$$

where $p^+ = (p^0 + p^1)/\sqrt{2}$ and $\tau = x^+ = (x^0 + x^1)/\sqrt{2}$. Here the superscripts 0 and 1 label the timelike and the longitudinal directions, and the transverse momentum $\mathbf{p}$ lives in the remaining D dimensions. The propagator is represented by a horizontal strip of width p^+ and length τ on the world sheet (Fig.1). The solid lines that form its boundary carry transverse momenta $\mathbf{q}_1$

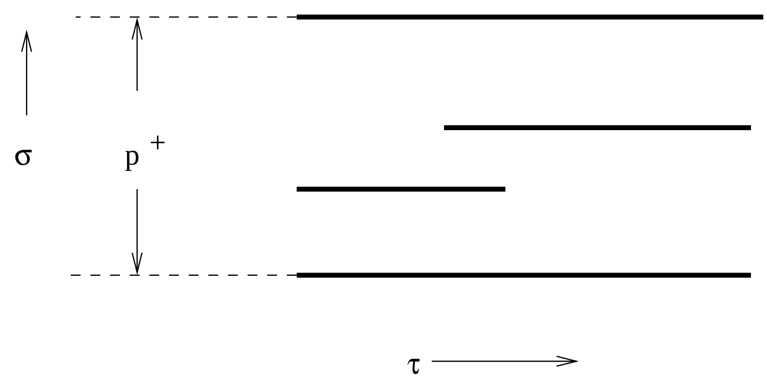

Figure 2: A typical graph

and $\mathbf{q}_2$ flowing in opposite directions so that

$$\mathbf{p} = \mathbf{q}_1 - \mathbf{q}_2.$$

More complicated graphs consist of several horizontal line segments (Fig.2). The ϕ^3 interaction takes place at the beginning and at the end of each segment, where a factor of g (the coupling constant) is inserted. One has then to integrate over the position of the vertices and over the momenta carried by the solid lines.

It was shown in [2,3] that the light cone Feynman rules sketched above can be reproduced by a local world sheet field theory. The world sheet is parametrized by the coordinate σ along the p^+ direction and τ along the x^+ direction, and the transverse momentum $\mathbf{q}$ is promoted to a bosonic field $\mathbf{q}(\sigma, \tau)$ on the world sheet. In addition, two fermionic fields b and c, each with the $D/2$ components (assuming that D is even), are needed. The action on the world sheet for the massless theory is given by

$$S_0 = \int_0^{p^+} d\sigma \int d\tau \left(b' \cdot c' - \frac{1}{2}\mathbf{q}'^2 \right), \tag{2}$$

where the derivative with respect to τ is represented by a dot and the derivative with respect to σ by a prime. Also, one has to impose the boundary conditions

$$\dot{\mathbf{q}} = 0, \quad b = c = 0, \tag{3}$$

on the solid lines. These boundary conditions can be implemented by introducing Lagrange multipliers $\mathbf{y}$, $\bar{b}$ and $\bar{c}$ and adding suitable terms to the

action. Since ghosts will not be needed in what follows, from now on, we will drop them. The action with the boundary conditions included, but without the ghosts, reads

$$S=\int_0^{p^+} d\sigma \int d\tau \left(-\frac{1}{2}\mathbf{q}'^2+\rho\,\mathbf{y}\cdot\dot{\mathbf{q}}\right). \tag{4}$$

Here the field ρ is a delta function on the boundaries and it vanishes in the bulk: it is inserted to ensure that the boundary condition is imposed only on the boundaries. However, with this insertion, the part of the integral over $\mathbf{y}$ that has support in the bulk diverges, since the integrand over this region is independent of $\mathbf{y}$. To avoid this problem, we add a Gaussian term to the action which cuts off the divergence:

$$\begin{aligned} S &\rightarrow S+S_{g.f}, \\ S_{g.f} &= \int_0^{p^+} d\sigma \int d\tau \left(-\frac{1}{2}\alpha^2\bar{\rho}\,\mathbf{y}^2\right), \end{aligned} \tag{5}$$

where α is a constant and $\bar{\rho}$ is complimentary to ρ: it vanishes on the boundaries and it is equal to one everywhere else. It was pointed out in [5,6] that this can be thought of as a gauge fixing term. In its absence, the action is invariant under the gauge transformation

$$\mathbf{y}\rightarrow\mathbf{y}+\bar{\rho}\,\mathbf{z},$$

where $\mathbf{z}$ is an arbitrary function of the coordinates. It may seem that we have introduced a new parameter α into the model, but we will see in section 6 that this new parameter can be absorbed into the definition of the cutoff parameters that will be needed shortly.

It is possible to give an explicit construction for the fields ρ and $\bar{\rho}$ in terms of a fermionic field on the world sheet. To see how this works, it is best to discretize the σ coordinate into segments of length a. This discretization is pictured in Fig.3 as a collection of parallel line segments, some solid and some dotted, spaced distance a apart. The boundaries are marked by the solid lines, and the bulk is filled with the dotted lines. Associated with these lines, there are a two component fermion field $\psi_i(\sigma_n,\tau)$ and also its adjoint $\bar{\psi}_i$, where, $\sigma_n=na$, is the discretized σ coordinate. The field $\bar{\psi}_1$ creates a dotted line and $\bar{\psi}_2$ a solid line out of vacuum, and $\psi_{1,2}$ annihilate these lines. ρ and $\bar{\rho}$ can now be written as

$$\rho=\frac{1}{2}\bar{\psi}(1-\sigma_3)\psi,\ \bar{\rho}=\frac{1}{2}\bar{\psi}(1+\sigma_3)\psi, \tag{6}$$

Figure 3: Solid and dotted lines

and the fermionic action is given by

$$\begin{aligned} S_f &= \sum_n \int d\tau \left(i\bar{\psi}\dot{\psi} - g\,\bar{\psi}\sigma_1\psi \right)_{\sigma=na} \\ &\to \int_0^{p^+} d\sigma \int d\tau \left(i\bar{\psi}\dot{\psi} - g\,\bar{\psi}\sigma_1\psi \right). \end{aligned} \tag{7}$$

Here, the first line represents the action in terms of the discretized fermions and the second line the corresponding continuum limit. The first term in the action corresponds to free fermion propagation along the τ direction, and the second term to interaction taking place when there is transition from a dotted line to a solid line or vice versa. We will work with both the discrete and the continuum pictures; the discrete version will be particularly useful in regulating the model. Thus the parameter a will serve as one of our two cutoffs. It is important to notice that there is a change in the normalization of the fermion field in passing from the discrete to the continuum picture. This is because there is a factor of a involved in converting a sum into an integral:

$$a\sum_n \to \int d\sigma,$$

which means that the fermion fields should be scaled as

$$\psi \to \frac{1}{\sqrt{a}}\psi, \quad \bar{\psi} \to \frac{1}{\sqrt{a}}\bar{\psi}, \tag{8}$$

in going over to the continuum normalization.

As we mentioned earlier, to reproduce all Feynman graphs, one has to sum (integrate) over all possible positions of the solid lines (boundaries). It can

readily been seen that summing over the two components of the fermion field at each point on the world sheet is equivalent to summing over all possible positions of the boundaries. Therefore, the introduction of the fermionic field enables one, at least formally, to sum over all planar Feynman graphs of the massless ϕ^3 theory, and the following world sheet action , gotten by adding the eqs.(4),(5) and(7), provides a compact expression for this sum:

$$S = \int_0^{p^+} d\sigma \int d\tau \left(-\frac{1}{2}\mathbf{q}'^2 + \rho\,\mathbf{y}\cdot\dot{\mathbf{q}} - \frac{1}{2}\alpha^2\bar{\rho}\,\mathbf{y}^2 + i\bar{\psi}\dot{\psi} - g\,\bar{\psi}\sigma_1\psi\right), \qquad (9)$$

where ρ and $\bar{\rho}$ are given by eq.(6).

There is, however, a problem with the above action: It fails to reproduce the prefactor $1/(2\,p^+)$ in eq.(1) for the propagator, unless, g, instead of being a constant, is allowed to depend on ρ. In [6], it was shown how to take this dependence into account within the meanfield approximation. We will ignore this problem, since we will see later that this complication does not arise in summing over strings.

Finally, we would like to discuss the behaviour of various fields under the scaling of coordinates, which is intimately connected with Lorentz invariance. In the light cone setup we have, the only Lorentz transformation that is still manifest is generated by the boost along the singled out direction labeled by 1. Under this boost, x^+ and p^+ scale as

$$x^+ \to u\,x^+, \;\; p^+ \to u\,p^+,$$

where u parametrizes the boost. If under this scaling, the fields transform as

$$\begin{aligned} \mathbf{q}(\sigma,\tau) &\to \quad \mathbf{q}(u\sigma,u\tau), \;\; \mathbf{y}(\sigma,\tau) \to \quad \mathbf{y}(u\sigma,u\tau), \\ \psi(\sigma,\tau) &\to \sqrt{u}\,\psi(u\sigma,u\tau), \;\; \bar{\psi}(\sigma,\tau) \to \sqrt{u}\,\bar{\psi}(u\sigma,u\tau), \end{aligned} \qquad (10)$$

then the action given by eq.(9) is invariant except for the interaction and the gauge fixing terms. These two terms become invariant, at least formally, only if we also require that

$$g \to u\,g, \;\; \alpha^2 \to u\,\alpha^2. \qquad (11)$$

It is somewhat unusual to require constants in an action to transform, and one may worry that Lorentz invariance is in danger. We will see later on how this problem is resolved.

This finishes the review of the massless ϕ^3 theory on the world sheet. We will not review the calculation of the ground state of this model in the mean field approximation given in reference [6], since in any case, the mean field method will be developed in the context summation of planar string graphs in the next sections.

3. String Summation

We start with the open bosonic string in the light cone picture, with $U(N)$ Chan-Paton factors. Taking the large N limit picks the planar graphs. For simplicity, the length of the world sheet in the τ direction is taken to be infinite, and periodic boundary conditions at $\sigma = 0$ and $\sigma = p^+$ are imposed, where p^+ is the total $+$ component of the momentum flowing into the world sheet. We will also use p^+ freely to denote the $+$ momentum flowing into individual strings; it will be clear from the context which p^+ is meant. The above setup has the advantage of being traslation invariant along both the σ and τ directions, which simplifies the subsequent calculations considerably. Fig.2, which pictured a ϕ^3 field theory graph, applies equally well a general planar open string graph, with the boundaries of individual propagating strings again marked by the solid lines (see, for example, [12]). Therefore, the fermionic action introduced in the last section for summing over the graphs of the ϕ^3 theory, works just as well for summing over planar open string graphs. Of course, there are some differences between field theory and string theory pictures. For example, the action S_0 for the free string propagator is now given by

$$S_0 = \int_0^{p^+} d\sigma \int d\tau \left(\frac{1}{2}\dot{\mathbf{x}}^2 - \frac{1}{2\beta^2}\mathbf{x}'^2 \right). \tag{12}$$

Comparing with eq.(2), we see that the transverse momentum $\mathbf{q}$ is replaced by the transverse position $\mathbf{x}$, and there an additional term involving a derivative with respect to time. We have also introduced an adjustable slope β/π since we are ultimately interested in the zero slope, or the field theory limit. Furthermore, the boundary condition on the solid lines is now different: The Dirichlet condition (eq.(3)) is replaced by the Neumann condition

$$\mathbf{x}' = 0. \tag{13}$$

The fermionic part of the action is unchanged. The total action is therefore

given by

$$S=\int_0^{p^+}d\sigma\int d\tau\left(\frac{1}{2}\dot{\mathbf{x}}^2-\frac{1}{2\beta^2}\mathbf{x}'^2+\rho\,\mathbf{y}\cdot\mathbf{x}'-\frac{1}{2}\alpha^2\bar{\rho}\,\mathbf{y}^2+i\bar{\psi}\dot{\psi}-g\,\bar{\psi}\sigma_1\psi\right). \tag{14}$$

Here, just as in eq.(9), the Lagrange multiplier $\mathbf{y}$ enforces the boundary conditions, and the term proportional to α^2 is inserted to cut off the divergent integral over $\mathbf{y}$ in the bulk. This world sheet action then provides a compact expression for the sum of planar open string graphs. We note that, unlike in the case of ϕ^3 field theory, where a prefactor in the propagator was missing , the string propagator of eq.(12) is exact. Therefore, the complication discussed following eq.(9) does not arise, and g can simply be taken to be a constant. An additional simplification is the absence of the ghost fields b and c (see eq.(2)). From this point of view, the sum over string graphs looks considerably simpler than the sum over field theory graphs.

In order to facilitate the comparison with field theory, we would like to convert (14) into a form as close to eq.(9) as possible. This will involve a duality transformation which makes the following interchanges:

$$\mathbf{x}\leftrightarrow\mathbf{q},\;\;\mathbf{x}'=0\leftrightarrow\dot{\mathbf{q}}=0. \tag{15}$$

The first step of the duality transformation is to integrate over $\mathbf{x}$ in eq.(14):

$$\begin{aligned} S &\rightarrow S_f+S_{g.f}+\frac{1}{2}iD\,Tr\ln\left(\beta^2\partial_\tau^2-\partial_\sigma^2\right)\\ &+\frac{\beta^2}{2}\int_0^{p^+}d\sigma\int d\tau\int_0^{p^+}d\sigma'\int d\tau'\,G(\sigma\tau,\sigma'\tau')\,\partial_\sigma(\rho\mathbf{y})_{\sigma\tau}\,\partial_{\sigma'}(\rho\mathbf{y})_{\sigma'\tau'}, \end{aligned} \tag{16}$$

where S_f and $S_{g.f}$ are given by eqs.(7) and (5), and the Green's function G satisfies

$$(\beta^2\partial_\tau^2-\partial_\sigma^2)G(\sigma\tau,\sigma'\tau')=\delta(\sigma-\sigma')\delta(\tau-\tau'). \tag{17}$$

We note that, because of the translation invariance on the world sheet, the Green's function depends only on the coordinate differences $\sigma-\sigma'$ and $\tau-\tau'$.

Next, we integrate the σ derivatives by parts, and use the translation invariance of the Green's function and the defining equation (17) to arrive at

$$S \;=\; S_f+S_{g.f}+\frac{1}{2}iD\,Tr\ln\left(\beta^2\partial_\tau^2-\partial_\sigma^2\right)+\frac{\beta^2}{2}\int_0^{p^+}d\sigma\int d\tau\rho^2\,\mathbf{y}^2$$

$$+ \quad \frac{\beta^4}{2}\int_0^{p^+} d\sigma \int d\tau \int_0^{p^+} d\sigma' \int d\tau' \, G(\sigma\tau, \sigma'\tau') \, \partial_\tau(\rho\mathbf{y})_{\sigma\tau} \, \partial_{\tau'}(\rho\mathbf{y})_{\sigma'\tau'}. \quad (18)$$

In order to have a sensible $\beta \to 0$ limit in this equation, we first scale $\mathbf{y}$ by

$$\mathbf{y} \to \mathbf{y}/\beta^2,$$

and rewrite it by introducing an auxiliary variable $\mathbf{q}$:

$$\begin{aligned} S &= S_f + S_{g.f} + \frac{1}{2\beta^2}\int_0^{p^+} d\sigma \int d\tau \rho^2 \, \mathbf{y}^2 \\ &+ \int_0^{p^+} d\sigma \int d\tau \left(\frac{1}{2}\beta^2 \dot{\mathbf{q}}^2 - \frac{1}{2}\mathbf{q}'^2 + \rho\, \mathbf{y} \cdot \dot{\mathbf{q}}\right). \end{aligned} \quad (19)$$

This action is quite similar to the world sheet action for ϕ^3 (eq.(9)) and in fact coincides with it in the zero slope limit $\beta \to 0$, except for the term

$$S' = \frac{1}{2\beta^2}\int_0^{p^+} d\sigma \int d\tau \, \rho^2 \, \mathbf{y}^2,$$

which blows up in this limit. We will now argue that this term should be absent. In fact, our starting point, eq.(14), was not quite correct; in the action S, the term S' should have been dropped. This point is perhaps made clearer by considering an electrostatic analogy. The Lagrange multiplier $\mathbf{y}$ which enforces $\mathbf{x}' = 0$ on the boundaries can be thought of as a line charge induced by the boundary condition. It is easy to see that S' is the (divergent) electrostatic self energy of the line charge in question. On the other hand, the Lagrange multiplier $\mathbf{y}$ was introduced solely to enforce the Neumann boundary conditions; the induced electrostatic energy is an additional boundary term which is absent in the usual treatment of the open string. It should therefore be dropped. As a further check, consider the configuration with two eternal boundaries at $\sigma = 0$ and $\sigma = p^+$: For this simple case, $\rho = \delta(\sigma) + \delta(\sigma - p^+)$ and, after scaling $\mathbf{y}$ by $1/\beta^2$, the last term in eq.(18) reduces to

$$S \to \frac{1}{2}\int_0^{p^+} d\sigma \int d\tau \int_0^{p^+} d\sigma' \int d\tau' \, G(\sigma\tau, \sigma'\tau') \, \partial_\tau(\mathbf{y})_{\sigma\tau} \, \partial_{\tau'}(\mathbf{y})_{\sigma'\tau'}. \quad (20)$$

The functional integral over $\mathbf{y}$ can be done, and after a simple calculation which we will not present here, the correct free propagator for the open string is reproduced. This justifies the dropping of S' from the world sheet action.

4. The Meanfield Calculation

The meanfield calculation we are going to present here is very similar to the treatment for the ϕ^3 theory given in [2-6]. There are, however, some important differences, which we will point out as we go along. The starting point is the world sheet action, derived at the end of the last section, which we write in full:

$$S = \int_0^{p^+} d\sigma \int d\tau \left(\frac{1}{2}\beta^2 \dot{\mathbf{q}}^2 - \frac{1}{2}\mathbf{q}'^2 + \rho\, \mathbf{y}\cdot\dot{\mathbf{q}} - \frac{1}{2}\alpha^2 \bar{\rho}\mathbf{y}^2 + i\bar{\psi}\dot{\psi} - g\,\bar{\psi}\sigma_1\psi \right). \tag{21}$$

Ultimately we will be interested in the $\beta \to 0$ (zero slope) limit, which will get us back to field theory, but we will study this limit only within the framework of the meanfield method. The reason for this indirect approach is that if one naively sets $\beta = 0$ in the above expression for S, one does not quite get the correct field theory result. For example, the ghost fields b and c are missing and also there is the problem of the missing prefactor in the propagator discussed following eq.(9). Later on, we will argue that the naive $\beta \to 0$ limit can be quite singular, but that a smooth limit can be defined within the framework of the mean field method.

It is convenient to view the mean field approximation as the large D limit of the field theory defined on the world sheet, where D is the number of transverse dimensions. We hasten to add that this is merely a convenient way of doing the correct bookkeeping; one can set D to any desired value at the end of the calculation. The idea is to cast the action into a form proportional to D and take the large D limit by the saddle point method. This is the standard way of solving for the anologous large N limit of so called vector models [13]. Following [5,6], we introduce the extra term ΔS in the action:

$$\begin{aligned} S &\to S + \Delta S, \\ \Delta S &= \int_0^{p^+} d\sigma \int d\tau \left(\kappa_1 (D\,\lambda_1 - \mathbf{y}\cdot\dot{\mathbf{q}}) + \frac{1}{2}\kappa_2 (D\,\lambda_2 - \mathbf{y}^2) \right). \end{aligned} \tag{22}$$

Integrating over $\kappa_{1,2}$, all we have done is to rename the composite fields $\mathbf{y}\cdot\dot{\mathbf{q}}$ and $\mathbf{y}^2$ as $D\lambda_1$ and $D\lambda_2$. The factors of D are natural since each composite field is the sum of D terms. The Gaussian integral over $\mathbf{y}$ is easily done, with the result,

$$S + \Delta S \to S_1 + S_2 + S_3,$$

$$
\begin{aligned}
S_1 &= \int_0^{p^+} d\sigma \int d\tau \left(\frac{1}{2}(\beta^2 + \kappa_1^2/\kappa_2)\,\dot{\mathbf{q}}^2 - \frac{1}{2}\mathbf{q}'^2\right), \\
S_2 &= D\int_0^{p^+} d\sigma \int d\tau \left(\kappa_1\lambda_1 + \frac{1}{2}\kappa_2\lambda_2\right), \\
S_3 &= \int_0^{p^+} d\sigma \int d\tau \left(i\bar{\psi}\dot{\psi} - g\,\bar{\psi}\sigma_1\psi + \frac{D}{2}(\lambda_-\bar{\psi}\psi - \lambda_+\bar{\psi}\sigma_3\psi)\right), \qquad (23)
\end{aligned}
$$

where, we have defined,

$$
\lambda_\pm = \pm\lambda_1 + \frac{1}{2}\alpha^2\lambda_2.
$$

Some of the terms in this equation can be further simplified. We observe that the operator $\bar{\psi}\psi$ represents the local fermion density. Since there is always one fermion on each horizontal line, independent of whether it is dotted or solid, one can set this operator equal to unity in the picture where the σ coordinate is discretized. On the other hand, in the continuum normalization, taking into account the scaling given by eq.(8), one can instead set

$$
\bar{\psi}\psi = 1/a. \qquad (24)
$$

After this substitution, λ_- becomes a Lagrange multiplier, enforcing the constraint

$$
\kappa_2 = \alpha^2(\kappa_1 + 1/a). \qquad (25)
$$

With these simplifications (eqs.(24) and (25)), the world sheet action becomes,

$$
\begin{aligned}
S &= \int_0^{p^+} d\sigma \int d\tau \Big(\frac{1}{2}A^2\dot{\mathbf{q}}^2 - \frac{1}{2}\mathbf{q}'^2 + \lambda(\kappa + 1/(2a)) \\
&+ i\bar{\psi}\dot{\psi} - Dg\,\bar{\psi}\sigma_1\psi - \frac{D}{2}\lambda\,\bar{\psi}\sigma_3\psi\Big), \qquad (26)
\end{aligned}
$$

where,

$$
A^2 = \beta^2 + \frac{\kappa^2}{\alpha^2(\kappa + 1/a)}, \qquad (27)
$$

and we have scaled the coupling constant by D,

$$
g \to D\,g, \qquad (28)
$$

in order to have an action that is proportional to D in the large D limit. Also, to simplify writing, we have set

$$
\kappa_1 = \kappa, \quad \lambda_+ = \lambda.
$$

It is important to note that after summing over strings, the slope parameter changed from β for the free string to the dynamical variable A. We will later compute the ground state expextation value of A, and show that it can differ from β.

Before closing this section, we would like to stress that the parameters so far introduced that define the model are in general cutoff dependent bare parameters. We already have one cutoff a, the spacing of the grid along the σ direction, and a', another grid spacing along the $\tau = x^+$ direction will soon be needed in order to regulate the integral over $\mathbf{q}$. How are the renormalized parameters, which should stay finite as the cutoffs are removed by letting

$$a \to 0, \quad a' \to 0,$$

to be defined? We will not address the problem of renormalization in any detail here[3], but one obvious condition is to demand that the renormalized parameters be invariant under the scaling transformation discussed at the end of the section 2. Since the scale transformation is the same as a special Lorentz transformation, this is clearly necessary for Lorentz invariance. The idea is then to define new scale invariant parameters by multiplying them with appropriate powers of a and a'. The slope β, which has the dimension of inverse mass squared, is already scale invariant. We also note that a and a' have the scaling properties (eq.(10)) and the dimensions of p^+ and x^+ respectively, so that the ratio

$$a/a' = m^2, \tag{29}$$

is scale invariant and has the dimension of mass squared. We will hold this ratio finite and fixed in the limit of a and a' going to zero. Therefore, there is only one cutoff, say a, and a mass parameter m in the problem. Eventually, we will consider the limit $\beta \to 0$ limit, and m will then be the only mass left in the model to set the mass scale.

In addition to β, a and a', there are two more constants in the problem: The coupling constant g and the gauge fixing parameter α. We trade them for scale invariant constants $\bar{g}$ and $\bar{\alpha}$ by defining

$$\bar{\alpha}^2 = \frac{\alpha^2 a'^2}{a}, \quad \bar{g} = \frac{g a'}{\pi}, \tag{30}$$

[3]See [14] for an investigation of renormalization and Lorentz invariance in the light cone formulation.

where the factor of π is introduced for later convenience. There was actually an ambiguity in the definition of the barred constants because of the availability of the scale invariant parameter m; we fixed this ambiguity by requiring $\bar{g}$ and $\bar{\alpha}$ to be dimensionless. We shall see in the next section that the slope of the interacting string is expressible in terms of these new constants, without any explicit dependence on the cutoff. This will provide some justification for calling them renormalized constants.

5. The Ground State Of The Model

In this section, the ground state of the model will be determined by minimizing the energy of the system. So far, everything has been exact: No approximations were made, for example, in deriving eq.(23). Of course, we are unable to do an exact calculation, so to make progress, we have to appeal to the large D limit. In this limit, the fields κ and λ are treated as classical quantities, to be calculated by the saddle point method. On the other hand, $\mathbf{q}$, ψ and $\bar{\psi}$ are still full quantum fields, to be integrated over functionally. In other words, in the leading large D limit, κ and λ are to be replaced by their ground state expectation values

$$\kappa_0 = \langle \kappa \rangle, \quad \lambda_0 = \langle \lambda \rangle.$$

In order to avoid excessive notation, from now on, we will drop the subscript zero, so that κ and λ will stand for the expectation values of these fields.

At this point, translation invariance along both the σ and τ directions comes in handy. It allows us set κ and λ equal to constants independent of the coordinates σ and τ. This means that A is also independent of the coordinates, and therefore the integrals over $\mathbf{q}$, ψ and $\bar{\psi}$ in the action S of eq.(26) can be done explicitly. Instead of evaluating a given action S directly, we find it more convenient to compute the corresponding energy E and express S in terms of E by means of

$$S = -i(\tau_f - \tau_i)E,$$

where $(\tau_f - \tau_i)$ is the (infinite) time interval. For example, the result of carrying out the integral over $\mathbf{q}$ in eq.(26) can be expressed as

$$S_1 = \frac{iD}{2}\, Tr \ln\left(A^2 \partial_\tau^2 - \partial_\sigma^2\right) = -i(\tau_f - \tau_i)E^{(0)}, \tag{31}$$

where $E^{(0)}$ is the zero point energy of the free string. Similarly, the result of doing the ψ and $\bar{\psi}$ integrations can be expressed in terms of the fermionic

energy E_f. Adding up, the total energy corresponding to S in eq.(26) is given by

$$E = E^{(0)} + E_f - p^+ \lambda\,(\kappa + 1/(2a)). \tag{32}$$

One point should be clarified here. What we have called the energy E is really the light cone energy p^-. Because of the periodic boundary conditions, the total transverse momentum is zero, and the invariant mass squared M^2 of the system is given by

$$M^2 = p^+\, E.$$

The next step is to evaluate $E^{(0)}$ and E_f. Since $E^{(0)}$ is well known from the standard calculation of the Casimir effect, we only remind the reader of the steps involved. The regulated zero point energy is given by

$$E^{(0)} = D \sum_0^\infty E_k\, \exp(-E_k/\Lambda), \tag{33}$$

where E_k is the zero point energy of the k'th SHO mode,

$$E_k = \frac{2\pi k}{p^+}, \tag{34}$$

and we have introduced an exponential regulator with the parameter Λ. The leading two terms in the limit $\Lambda \to \infty$ are given by

$$E^{(0)} \to D\left(\frac{A\,\Lambda^2\, p^+}{2\pi} - \frac{\pi}{3A\, p^+}\right). \tag{35}$$

Of course, any other smooth regulator that depends only on E_n gives the same result.

The regulator Λ acts as a cutoff in energy; it is related to a', the spacing of the grid in the conjugate variable τ, by

$$a' = \frac{2\pi}{\Lambda},$$

and replacing Λ by a' in eq.(35) gives

$$E^{(0)} = 2\pi D\, p^+ A/(a')^2, \tag{36}$$

where we have kept the cutoff dependent term and dropped the finite one. In calculating the Casimir effect, one does the opposite: The cutoff dependent

term is subtracted and the finite term is kept. Here, this term, through A, depends on the dynamical variable κ (eq.(27)), and there is no way to cancel it by introducing a constant counter term, independent of the dynamical variables. In fact, since the cutoff dependent term dominates over the finite term, we have dropped the latter.

The fermionic energy E_f is evaluated by diagonalizing the corresponding Hamiltonian

$$
\begin{aligned}
H_f &= \sum_n H^{(n)}, \\
H^{(n)} &= D\left(\frac{1}{2}\lambda\,\bar{\psi}\sigma_3\psi + g\,\bar{\psi}\sigma_1\psi\right)_{\sigma=na}, \qquad (37)
\end{aligned}
$$

which has been regulated by discretizing σ. We have already remarked that λ (and of course, also g) is a constant, and therefore, $H^{(n)}$ reduces to a constant two by two matrix in the two dimensional space spanned by $\bar{\psi}_i$:

$$
H^{(n)} \to \begin{pmatrix} \lambda/2 & g \\ g & -\lambda/2 \end{pmatrix}. \qquad (38)
$$

The two eigenvalues of this matrix ,

$$
E_{\pm} = \pm\frac{1}{2}\sqrt{\lambda^2 + 4g^2},
$$

have to be multiplied by the number of points forming the σ grid, p^+/a, in order to obtain the total fermionic energy:

$$
E_{f,\pm} = \frac{p^+}{2a}\sqrt{\lambda^2 + 4g^2}. \qquad (39)
$$

Notice that the fermionic energy has two possible values. Clearly, the choice $(-)$ corresponds to the ground state, but we will also be interested in the other possibility.

Putting together eqs.(32),(35) and (39), we have the following expression for the total energy:

$$
\begin{aligned}
E_{\pm} &= Dp^+\Bigg(\frac{2\pi}{a'^2}\sqrt{\beta^2 + \frac{\kappa^2}{\alpha^2(\kappa + 1/a)}} \pm \frac{1}{2a}\sqrt{\lambda^2 + 4g^2} \\
&- \frac{1}{2}\lambda(2\kappa + 1/a)\Bigg). \qquad (40)
\end{aligned}
$$

As explained earlier, we are looking for the saddle point of this expression in the variables λ and κ. The saddle point satisfies,

$$\frac{\partial E_\pm}{\partial \lambda} = 0, \quad \frac{\partial E_\pm}{\partial \kappa} = 0.$$

The first equation determines λ:

$$\lambda = \pm g \frac{1 + 2a\kappa}{\sqrt{-a\kappa - a^2\kappa^2}},$$

and using this result, the λ dependence of the energy can be eliminated, leaving it as a function of only κ. Before writing down the result, it is convenient to replace κ by a cutoff independent and dimensionless new variable x through

$$x = -a\kappa, \tag{41}$$

and the constants α and g by their cutoff independent counterparts $\bar{\alpha}$ and $\bar{g}$ through eq.(30). After these substitutions, the expression for the energy is given by

$$E_\pm = \frac{2\pi D\, p^+}{a a'} \left(\sqrt{\beta^2 m^4 + x^2/(1-x)} \pm \bar{g}\sqrt{x - x^2} \right). \tag{42}$$

We pause briefly to discuss the physical significance of x. By computing the eigenvectors of the matrix (38), it is easy to show that [6],

$$\frac{1}{2}\langle\bar{\psi}(1 - \sigma_3)\psi\rangle = \langle\rho\rangle = x, \quad \frac{1}{2}\langle\bar{\psi}(1 + \sigma_3)\psi = \langle\bar{\rho}\rangle = 1 - x, \tag{43}$$

where $\langle\rangle$ represents the ground state expectation value. This is in the discretized version of the world sheet; in the continuum version, x and $1-x$ should be replaced by x/a and $(1-x)/a$. Therefore, in the discrete version, x is the average probability of finding a spin down fermion on the world sheet. By the definition, this is the same as the average probability of finding a solid line. Conversely, $1-x$ is the average probability of finding a dotted line. From this probability interpretation, it is clear that

$$0 \le x \le 1. \tag{44}$$

We should emphasize that, for the probability to be well defined, it was important to have a discretized world sheet, with the grid spacing a kept fixed.

The next step is to find the minima of $E_\pm$ as a function of x. This is easy in the case of E_+: It has a minimum at $x = 0$, with the value

$$E_+ = \frac{2\pi D\, p^+ \beta m^2}{aa'}. \tag{45}$$

The true minimum is, of course, the minimum of E_-. The value of x_m that minimizes E_- cannot be found analytically, but one can get approximate answers in the two interesting limits: $\bar{g} \ll 1$ (weak coupling), and $\bar{g} \gg 1$ (strong coupling). We have also to distinguish between two cases, depending on whether the initial slope is non-zero ($\beta m^2 \neq 0$), or it is zero ($\beta m^2 = 0$). Taking $\beta m^2 \neq 0$ and $\bar{g} \ll 1$, to leading order in $\bar{g}$, the minimum is given by,

$$\begin{aligned} x_m &\approx \left(\frac{\beta m^2 \bar{g}}{2}\right)^{2/3}, \\ E_{-,m} &\approx \frac{2\pi D\, p^+}{aa'}\left(\beta m^2 - (2)^{-1/3}(\bar{g})^{4/3}(\beta m^2)^{1/3}\right). \end{aligned} \tag{46}$$

We see that, in the weak coupling limit, x_m is small and the minimum of E_- is less then the minimum of E_+, as expected. On the other hand, in the strong coupling limit, $\bar{g} \gg 1$, x_m approaches $1/2$:

$$\begin{aligned} x_m &\approx 1/2 - \frac{3}{4\bar{g}}\left(\beta^2 m^4 + 1/2\right)^{-1/2}, \\ E_{-,m} &\approx -\frac{\pi D\, p^+}{aa'}\bar{g}. \end{aligned} \tag{47}$$

Now let us consider the case of zero slope for the free string, $\beta m^2 = 0$. in the weak coupling limit, the minimum is given by

$$x_m \approx \bar{g}^2/4, \quad E_{-,m} \approx -\frac{\pi D\, p^+}{2aa'}\bar{g}^2, \tag{48}$$

and in the strong coupling limit by

$$x_m \approx 1/2 - \frac{3}{2\sqrt{2}\bar{g}}, \quad E_{-,m} \approx -\frac{\pi D\, p^+}{2aa'}\bar{g}^2. \tag{49}$$

From the above results, it is clear that the cases of non-zero and zero initial slope are qualitatively similar. In both cases, x_m ranges from zero to $1/2$ as the coupling constant $\bar{g}$ varies from zero to infinity. The function,

$$f(x) = \frac{aa'}{2\pi D\, p^+}\, E_- = \sqrt{\beta^2 m^4 + x^2/(1-x)} - \bar{g}\sqrt{x - x^2},$$

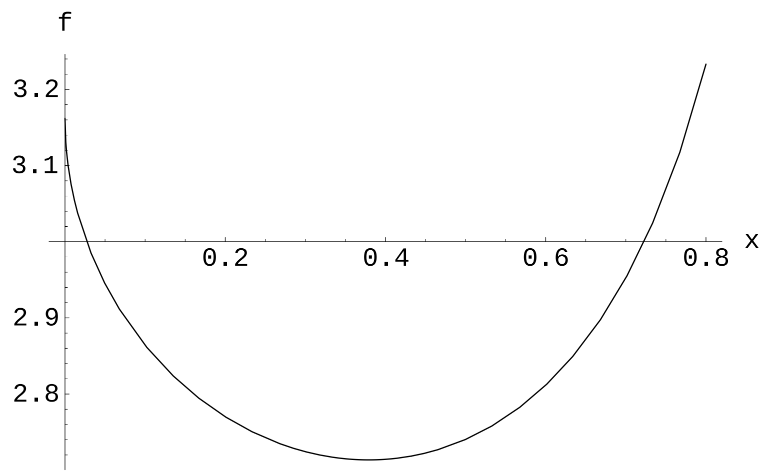

Figure 4: The plot of $f(x)$ against x for $\beta m^2 = 10$ and $\bar{g} = 1$

is plotted against x for $\beta m^2 = 10$, $\bar{g} = 1$ and for $\beta m^2 = 0$, $\bar{g} = 1$, in Figs.4 and 5 respectively.

6. Discussion Of The Results

In the last section, we have seen that:
1) There are two saddle points of the model, with ground state energies E_+ and E_-. The true ground state of the model corresponds to E_-, which is always less than E_+. We will call the first solution the $(+)$ phase and the second one the $(-)$ phase.
2) The minimum of E_+ is realized at $x_m = 0$, whereas the minimum of E_- is at some value of x_m that satisfies

$$0 < x_m < 1/2.$$

3) These statements are true for both finite initial slope β, and also in the limit $\beta \to 0$.

We argued in the last section, following eq.(43), that x represents the average probability of finding a solid line on the discretized world sheet. Since the solid lines represent the boundaries, in effect, x measures the average density of the string boundaries. For the perturbative sum over strings that we started with, we expect x_m to be zero, since at each order of perturbation, the boundaries form a set of measure zero. It is then natural to identify the solution corresponding to the $(+)$ phase, whose minimum is at $x_m = 0$, with

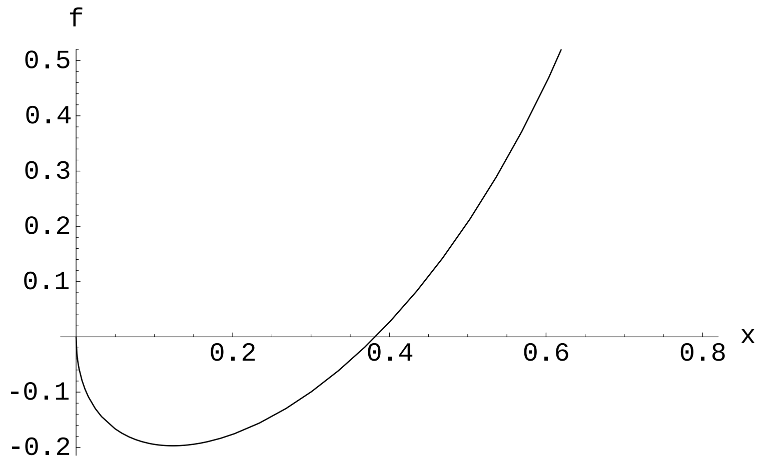

Figure 5: The plot of $f(x)$ against x for $\beta m^2 = 0$ and $\bar{g} = 1$

the perturbative string sum. In contrast, in the $(-)$ phase, where $x_m \neq 0$, the boundaries form a dense set on the world sheet. We will call this phenomenon, which cannot be realized in any finite order of perturbation, the condensation of string boundaries. The order parameter that distinguishes between these two phases is x, the expectation value of the composite operator ρ (eq.(43)). The statements made above are quite general, independent of any approximation scheme. Without a dynamical calculation, however, we do not know which of the phases has the lower energy. We have seen above that, at least in the mean field approximation, the $(-)$ phase, in which the boundaries have condensed, has the lower ground state energy.

At the end of the last section, we studied the dependence of x_m in the $(-)$ phase on the coupling constant $\bar{g}$, and found that $x_m \neq 0$ for all non-zero $\bar{g}$. It is at first surprising that condensation of boundaries takes place even at small $\bar{g}$, but we have to remember that the original coupling constant g has already been scaled by a factor of D (eq.(28)). The mean field approximation is based on the limit $D \to \infty$, and in this limit, even small values of $\bar{g}$ may give rise to large values of g. On the other hand, it is easy to understand what happens for large $\bar{g}$. In this limit, the fermionic energy E_f (second term in eq.(42)), which is proportional to $\bar{g}$, dominates, and the minimum of this term is at $x_m = 1/2$. The limiting value of $x_m = 1/2$ is also easy to understand: The large coupling constant limit energetically favors a maximum density of string vertices. Since vertices convert a solid line into a dotted line and vice

versa, it is advantageous to flip between solid and dotted lines as often as possible. It is easy to see that this corresponds to an equal density for the solid and dotted lines, namely, $x = 1/2$.

It was pointed out earlier that, after the summation over free strings is carried out, the free string slope β/π is replaced by the A/π . This is a fluctuationg dynamical variable, but we could define an average slope in terms of the ground state expectation value of A. Replacing α and κ in eq.(27) by $\bar{\alpha}$ and x through (30) and (41) gives

$$\langle A^2 \rangle = \frac{1}{m^4} \left(\beta^2 m^4 + \frac{x_m^2}{\bar{\alpha}^2 (1 - x_m)} \right). \tag{50}$$

For the (+) solution with $x_m = 0$, the average slope after the summation is the same as the free string slope, which is consistent with the perturbative picture. On the other hand, the (−) solution, with $x_m \neq 0$, gives a slope larger than the free slope. So what emerges is a new string with a larger slope, and also, of course, with a more complicated structure. We can also see from the above equation that $\bar{\alpha}$ is a redundant parameter; one can absorb it into the definition of m by redefining one of the cutoff parameters a or a'.

To show that A becomes a genuine dynamical variable, one has to go to next to leading order in the large D limit. We will not go into the details here, since the calculation is almost identical to the analogous one in the case of field theory, given in references [5,6]. The end result is that, a kinetic energy term in the action,

$$S^{(2)} = \frac{D \ln(2\pi p^+/a')}{8\pi \left(\langle A^2 \rangle\right)^{3/2}} \int_0^{p^+} d\sigma \int d\tau \left(\langle A^2 \rangle \left(\partial_\tau A\right)^2 - \left(\partial_\sigma A\right)^2 \right), \tag{51}$$

for A is generated, and so this variable becomes truly dynamical. What happens here is quite similar to what happens in some other two dimensional models: An auxiliary classical variable acquires kinetic energy term and becomes truly dynamical when one loop contributions are taken into account [16,17].

Let us now consider the limit $\beta \to 0$. As noted earlier, this limit is rather delicate in the (+) phase; for example, naively letting $\beta \to 0$ directly in eq.(21) is not correct. It is easy to spot the problem: In this phase, as β and x_m go to zero, so does A, and at $A = 0$, the mean field method is no longer applicable. What happens is that the non-leading terms tend to become singular. For example, the second term in eq.(35) for the zero point energy,

which was neglected compared to the first term, blows up at $A = 0$. Also, non-leading order terms in the large D expansion of S_1 (eq.(31)) become singular in the same limit, invalidating the expansion. This can perhaps be guessed by setting $A = 0$ directly in S_1; the resulting expression loses its τ dependendence and becomes ill defined. In contrast, in the $(-)$ phase, A stays finite as $\beta \to 0$, and the non-leading terms in the meanfield (large D) expansion remain well-defined.

Summarizing the foregoing discussion, we conclude that, in the limit of the initial slope tending to zero:
1) The induced slope A also goes to zero in the $(+)$ phase, which causes the breakdown of the mean field method.
2) In contrast, A remains non-zero in the $(-)$ phase, and there are no obvious problems with the mean field method. Since $(-)$ is the energetically favored phase, we believe that this is actually what happens.
3) The string slope becomes a fluctuating dynamical variable (eq.(51)).

The zero slope limit is commonly thought of as the field theory limit. As $\beta \to 0$, the massive particles decouple, and one is left with a field theory built out of the massless particles. If we accept this picture, it follows that the summation of the field theory graphs has led to string formation. The sequence of the steps in the reasoning is the following: Start with the sum of planar open strings, and then take the zero slope limit. Order by order in the perturbation expansion, we expect the string graphs to tend to the field theory graphs. However, this is a limit rather difficult to define cleanly in mathematical terms because of the existence of the tachyon, whose mass squared goes to $-\infty$ in this limit, if the vector meson mass is fixed at zero. If, however, instead of first taking the zero slope limit, one reverses the order of the steps by first summing over strings and then taking the zero slope limit, the mean field calculation goes through smoothly, and the final induced slope is non-zero, signaling the formation of a new string.

7. Conclusions And Future Directions

In this article, we have applied the mean field method to the sum of planar open bosonic string diagrams on the world sheet. After a duality transformation, the problem was cast in a form very similar to the problem of summing planar ϕ^3 graphs [2-6], and the techniques developed earlier could be applied here. The results were also similar: The ground state of the system turned out to be a condensate of the open string boundaries. As a result, a

new string was formed, with a slope greater than the initial slope. Even in the limit of vanishing initial slope, the final slope remained non-zero.

We end by listing some remaining open problems. We would like to identify the zero slope limit of the initial string with the field theory of the zero mass vector particle, but the existence of the tachyon makes this identification problematic. A future project is to apply the methods developed here to the tachyon free superstrings. Another problem is the cutoff dependence of the ground state energy given by eq.(42). In reference [6], a similar cutoff dependence was cancelled by introducing a bare mass term for the ϕ field. Since here our starting point is already a string theory, we do not have this freedom initially. We could, nevertheless, introduce a counter term at the end. This then brings up the question of the finite part of the ground state energy left over after the cancellation of the cutoff dependence. We remind the reader that in the usual treatment of the bosonic string, this finite part is related to the intercept and it is not arbitrary. In the lightcone formulation, it is determined by requiring Lorentz invariance [15]. This brings up another important open problem; namely, the Lorentz invariance of the string that emerges after the condensation of the boundaries. If the intercept of this new string is determined by Lorentz invariance, this would shed light on the question of tachyon condensation in the open string [10]. We hope to address at least some of these problems in the future.

Note Added:

After finishing this paper, reference [18] was brought to my attention. There is considerable overlap between this reference and the present article. Both pieces of work tackle the problem of summing planar strings on the world sheet using the mean field approximation, and both find condensation of boundaries and a new string with increased slope. There are, however, several important differences in the treatment of the problem. For example, in this article, unlike in [18], the string boundary conditions are imposed by means of Lagrange multipliers, and before applying the mean field approximation, Neumann conditions are replaced by Dirichlet conditions by means of a duality transformation. Also, some of our conclusions differ: Reference [12] finds a free string at the end of the summation, whereas we find a more complicated string with a fluctuating dynamical slope. Furthermore, in contrast to [12], starting with zero slope (field theory), we find string formation. These differences in the final result are probably due to the differences in the

treatment of the problem mentioned above.

Acknowledgements

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